

CALCULUS

CALCULUS COURSE OUTLINE

- I. ELEMENTARY & TRIGONOMETRIC FUNCTIONS
- II. LIMITS & CONTINUITY
- III. DERIVATIVES
- IV. INTEGRATION
- V. VECTORS
- VI. SEQUENCES & SERIES
- VII. TRANSCENDENTAL FUNCTIONS
- VIII. CONIC SECTIONS & POLAR COORDINATES

CALCULUS COURSE RATIONALE

At the end of Calculus, students will be able to develop abilities in mathematics to accomplish the following major goals:

1. Demonstrate the ability to identify and apply properties of elementary functions.
2. Demonstrate the ability to apply the rules of differentiation to find derivatives of elementary functions.
3. Demonstrate the ability to apply the theory of limits and continuity.
4. Demonstrate the ability to apply the concepts of differentiation.
5. Demonstrate the ability to find the general antiderivative of an elementary function.
6. Demonstrate the ability to find the definite integral of an elementary function.
7. Demonstrate the ability to apply the concepts of integration.
8. Demonstrate the ability to apply the concepts of vectors.
9. Demonstrate the ability to apply the knowledge of sequences and series.
10. Demonstrate the ability to identify and apply the properties of transcendental functions.
11. Demonstrate the ability to identify and apply the properties of conic sections and polar coordinates.

CALCULUS INSTRUCTIONAL OBJECTIVES

Students will be able to:

1. Analyze functions by:
 - a. Identify the increasing, decreasing or constant intervals.
 - b. Find any relative maximum or minimum values of the function.
 - c. Specify the domain and range.
2. Find limits using a variety of methods.
3. Use limits to determine continuity.
4. Apply the definition of derivatives
5. Calculate derivatives using the Product, Sum, Quotient and Chain Rules.
6. Find derivatives of higher order.
7. Find the derivatives of an inverse
8. Find derivatives by special methods
9. Apply derivatives
10. Use the properties of definite integrals
11. Using antiderivatives, evaluate each of the following, then check your answer using technology
12. Apply Integrals
13. Compute magnitude, sum, difference, dot product and angle between vectors \vec{u} and \vec{v}
14. Find parallel and perpendicular vectors
15. Find the angle between two three-dimensional vectors
16. Find sums, lengths, scalar products, dot products and cross product of three-dimensional vectors.
17. dimensional vectors.
18. Describe sequences and find limits

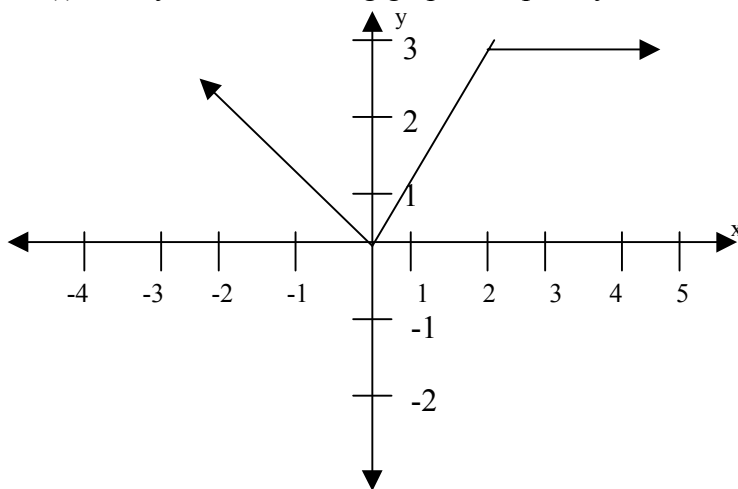
19. Describe series
20. Find sums of series
21. Understand and use Maclaurin series
22. Understand and use transcendental functions with derivatives
23. Understand and use transcendental functions with integrals
24. Find polar coordinates for a variety of figures

LEARNING OUTCOMES FOR CALCULUS

Students will be able to:

1. Analyze functions by:
 - a. Identify the increasing, decreasing or constant intervals.
 - b. Find any relative maximum or minimum values of the function.
 - c. Specify the domain and range.

Ex. (i) Analyze the following graph completely:



(ii) Given the function $f(x) = \frac{1 + \cos 2x}{2}$

- a. How large does $\cos 2x$ become? How small?
- b. Find the amplitude period and phase shift of f
- c. What are the domain and range of f

(iii) a. Find the zeros of the polynomial function

$$f(x) = 12x^3 - 65x^2 + 74x - 24$$

b. Find the zeros of the polynomial function

$$f(x) = 6x^4 - 25x^3 + 24x^2 + 2x - 3$$

c. Use the zoom feature of a graphing calculator to estimate, to the nearest hundredth, the zeros of the polynomial function

$$f(x) = x^3 - 3x^2 + 2x - 5$$

2. Find limits using a variety of methods.

Ex. Find a) $\lim_{x \rightarrow 3} \frac{x^2 + 9}{x^2 + 4}$

d) If $f(x) = \begin{cases} x & \text{if } x < 1 \\ 2 & \text{if } x = 1 \\ 1 & \text{if } x > 1 \end{cases}$ $\lim_{x \rightarrow 1^-} f(x)$

b) $\lim_{x \rightarrow \pi} \frac{x + 3}{\cos x}$

e) $\lim_{x \rightarrow \infty} \frac{2x^2 + x - 3}{x^3 + 4x + 2}$

c) $\lim_{x \rightarrow 4} \frac{\sqrt{x - 2}}{x - 4}$

3. Use limits to determine continuity.

Ex. (i) a) Graph the function $f(x) = \begin{cases} 0 & \text{if } x < 1 \\ 1 & \text{if } 0 \leq x < 1 \\ 0 & \text{if } 1 < x \end{cases}$

b) Determine at what points is the function continuous or discontinuous

(ii) Verify the Intermediate Value Theorem for $f(x) = x^3 + 1$ over $[-1, 2]$ for $f(x) = 2$

4. Apply the definition of derivatives

Ex. a) $f(x) = 10 - 3x^4$

b) $f(x) = \frac{x}{x + 1}$

c) $f(x) = \frac{1}{\sqrt{4 - x}}$

d) $f(x) = \sin x^2$

5. Calculate derivatives using the Product, Sum, Quotient and Chain Rules.

Ex. a) $f(x) = 7x^4 - 3x^3 + 2x - 1$ d) $f(x) = [\sin(x^2 - 1)]^3$

b) $f(x) = x^2 \cos x$

e) $g(x) = \sqrt[3]{x^2 + 4}$

c) $f(x) = \frac{4x - 3}{x^2 + 4}$

f) $f(x) = \ln \frac{3 + 4x}{x^2 - 5}$

6. Find derivatives of higher order.

Ex. Find:

- a) $f'''(x)$ if $f(x) = e^{2x}$
- b) $g''(x)$ if $g(x) = \sin^2 x$
- c) $\frac{d^2}{dx^2} \ln(2 + \cos x)$

7. Find the derivatives of an inverse

- Ex. (i) if $f(x) = 5x^2$ and g is the inverse of f , evaluate $g'(5)$
(ii) find $\frac{d}{dx} \sin^{-1} x^2$

8. Find derivatives by special methods

- Ex. (i) Use l' Hopital's Rule to evaluate $\lim_{x \rightarrow 0} \frac{\tan 3x}{\sin x}$
(ii) Apply the Mean Value Theorem to $f(x) = x^2 + x + 1$, if $a = 0$ and $b = 1$, find the value of c so that $\frac{f(b) - f(a)}{b - a} = f'(c)$

9. Apply derivatives

- Ex. (i) For $f(x) = \frac{x^3}{3} - \frac{x^2}{2} + 4$ Find:
a) the first derivative
b) the critical points
c) the intervals where f is increasing and decreasing
d) where there is a maximum and where there is a minimum
e) the second derivatives and determine the concavity and points of inflection then sketch the graph of f .
- (ii) The number of gallons of water in a tank t minutes after the tank has started to drain is $Q(t) = 200(30 - t)^2$. How fast is the water running out after 5 minutes?

10. Use the properties of definite integrals

- Ex. Given $\int f(x) dx = 5$, $\int g(x) dx = -3$ and $\int f(x) dx = 10$
Find the value of:
a) $\int [2f(x) + g(x)] dx$
b) $\int -g(x) dx$
c) $\int f(x) dx$
d) $\int [3g(x) - 4f(x)] dx$

11. Using antiderivatives, evaluate each of the following, then check your answer using technology

Ex. (i) $\int x^3(1 - x^4)^5 dx$

(ii) $\int x^2 \sin(2x) dx$

(iii) $\int \frac{x^3}{\sqrt{9-x}} dx$

(iv) $\int \sin^3 x \cos^3 x dx$

(v) $\int \frac{x^2 + 13x - 5}{2x^2 + 5x - 3} dx$

12. Apply Integrals

Ex.

(i) Find the velocity and position functions if the acceleration function is

$$a(t) = -32 \sin(2t), v(0) = 16 \text{ and } s(0) = 2$$

(ii) If the slope of the tangent line to the curve $y = f(x)$ is given by the slope $13x^2 - y$

where x and y are positive, find y as a function of x .

(iii) The amount of a certain radioactive substance is 50g at 9:00 a.m. At 11:00 a.m. that same day, the amount is 35g. How many grams of the substance were there at 8:30 a.m. the day the experiment began?

(iv) Write the necessary integral(s) and analytically evaluate the antiderivatives that would find the area in quadrant I bounded by $y = x - x^3$

13. Compute magnitude, sum, difference, dot product and angle between vectors \vec{u} and \vec{v}

Ex. Let $\vec{u} = (3, -4)$ and $\vec{v} = (-5, 2)$ and compute

a) $|\vec{v}|$

b) $\vec{u} - \vec{v}$

c) $-2\vec{u} + 5\vec{v}$

d) $\vec{u} \cdot \vec{v}$

e) the angle between \vec{u} and \vec{v}

f) are the vectors orthogonal?

14. Find parallel and perpendicular vectors

Ex. (i) Let $\vec{v} = (-2, 7)$. Find a vector parallel to \vec{v} .
(ii) Let $\vec{w} = (4, 1)$. Find a vector perpendicular to \vec{w} .

15. Find the angle between two three-dimensional vectors

Ex. Find the measure of the angle between the vectors:

$$\vec{u} = (-5, 2, 1) \text{ and } \vec{v} = (-2, -3, 4)$$

16. Find sums, lengths, scalar products, dot products and cross product of three-dimensional vectors.

Ex. Let $\vec{u} = (-5, 2, 1)$ and $\vec{v} = (-2, -3, 4)$. Compute each of the following:

$$\begin{aligned} \text{(a) } & \vec{u} + \vec{v} & \text{(b) } & \vec{u} - \vec{v} & \text{(c) } & \vec{u} \cdot \vec{v} \\ \text{(d) } & 2\vec{u} - 3\vec{v} & \text{(e) } & |\vec{u}| \end{aligned}$$

17. Describe sequences and find limits

Ex. Which of the sequences $\{a_n\}$ below converge and which diverge? Find the limit of each convergent sequence.

a) $a_n = \frac{n^2 - 2n + 1}{n - 1}$

b) $a_n = \frac{\sin n}{n}$

18. Describe series

Ex. Which of the series below converge absolutely, which converge conditionally, and which diverge? Give reasons for your answers.

a) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

b) $\sum_{n=1}^{\infty} \frac{1}{2n^3}$

19. Find sums of series

Ex. Find the sum of the series below

a)
$$\sum_{n=2}^{\infty} \frac{1}{(2n-3)(2n-1)}$$

b)
$$\sum_{n=0}^{\infty} e^{-n}$$

20. Understand and use Maclaurin series

Ex. Find Maclaurin series for the function

a)
$$\frac{1}{1-2x}$$

b)
$$\sin \frac{2x}{3}$$

21. Understand and use transcendental functions with derivatives

Ex. Find the derivative of y with respect to the appropriate independent variable

a) $y = 8^{-t}$ b) $y = \log_2 \frac{x^2}{2}$

22. Understand and use transcendental functions with integrals

Ex. Evaluate the integrals

a)
$$\int \frac{dx}{3x-4}$$

$$\int \frac{3}{x} dx$$

23. Find polar coordinates for a variety of figures

Ex. a) Find the polar coordinates of the point P(2, $\mu/6$)

b) Replace the Cartesian equations below by equivalent polar equations:
 $x^2 + y^2 = 4$

c) Find the points of intersection of the curves $r^2 = 4 \cos \theta$ and $r = 1 - \cos \theta$

Calculus Checklist	M- Mastered NI- Needs Improvement
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